Re-exam Advanced Logic

July 6th, 2016

Instructions:

- Put your student number on the first page and subsequent pages (not your name, in order to allow for anonymous grading).
- Do not use pencil (or an erasable pen) or a red pen to make your exam.
- Motivate all your answers.
- Your exam grade is computed as min(10, (the sum of all your points + 10) divided by 10). For example, someone who would only fill in a student number would get a 1, while someone who would make questions 1-9 perfectly but would skip the bonus question would get min(10, (90+10)/10) = 10.
- If you want to inspect your exam after it has been graded, you can do so by making an appointment with Rineke Verbrugge.

Good luck!

1. **Induction (10 pt)** Consider the sublanguage $\mathscr{L}_{\neg\vee}$ of the language of propositional logic with $\neg$ and $\vee$ as its only logical operators. (So without $\wedge$, $\rightarrow$ and $\leftrightarrow$).

   (a) Consider also the inductively defined operator n on well-formed formulas A of this language, which is defined as follows:

   - $n(A) = 1$ if A is a propositional parameter
   - $n(\neg A) = -1 * n(A)$
   - $n(A \vee B) = n(A) * n(B)$

   If $n(P) = 1$ for some well-formed formula of $\mathscr{L}_{\neg\vee}$, what does this tell us about P? Illustrate this using the formulas $\neg\neg\neg a \vee \neg b$ and $a \vee \neg(b \vee c)$.

   (b) Consider also the sublanguage $\mathscr{L}_{\rightarrow\wedge}$ of the language of propositional logic with $\rightarrow$ and $\wedge$ as its only logical operators.

   i. Inductively define an operator $'$ on well-formed formulas of $\mathscr{L}_{\rightarrow\wedge}$ such that for each well-formed formula A of $\mathscr{L}_{\rightarrow\wedge}$, the formula A' is a well-formed formula of $\mathscr{L}_{\neg\vee}$ which is logically equivalent to A. (In this part (b) i you do not have to prove this, just to define the right operator $'$; the proof of logical equivalence will be (b) ii.)

   ii. For the operation $'$ that you defined in (b) i, prove by induction that for each well-formed formula A of $\mathscr{L}_{\rightarrow\wedge}$ and for all valuations v, we have $v(A) = v(A')$.
   (So that indeed, for each well-formed formula A of $\mathscr{L}_{\rightarrow\wedge}$, the formula A' in $\mathscr{L}_{\neg\vee}$ is logically equivalent to A.)

2. **Three-valued logics (10 pt)** Using a truth table, determine whether the following inference holds in $\mathbf{K}_3$:

$$p \supset (p \wedge q) \models_{K_3} (\neg p \vee q) \wedge p$$

   Do not forget to draw a conclusion.

3. **Tableaus for FDE and related many-valued logics (10 pt)**

   (a) By constructing a suitable tableau, determine whether the following inference is valid in **LP**. If the inference is invalid, provide a counter-model.

$$((p \wedge q) \supset r) \wedge ((q \wedge r) \supset s) \vdash_{LP} (p \wedge q) \supset s$$

   NB: Do not forget to draw a conclusion from the tableau.

   (b) Is the inference valid in $\mathbf{K_3}$? If so, explain why. If not, provide a counter-model.

4. **Fuzzy logic (10 pt)** Determine whether the following holds in the fuzzy logic with $D = \{x : x \geq 0.7\}$. If so, explain why. If not, provide a counter-model and explain why not.

$$p \to (q \wedge r) \models_{0.7} (p \to q) \wedge (p \to r)$$

5. **Basic modal tableau (10 pt)** By constructing a suitable tableau, determine whether the following is valid in K. If the inference is invalid, provide a counter-model.

$$\Diamond p \vdash (\Box\neg p \supset \Box(p \supset q)) \wedge (\Box p \supset \Diamond(q \supset p))$$

NB: Do not forget to draw a conclusion from the tableau.

6. **Normal modal tableau (10 pt)** By constructing a suitable tableau, determine whether the following tense-logical inference is valid in K^t_ρ. If the inference is invalid, provide a counter-model.

$$\langle P\rangle(r \vee [P]q) \vdash_{K^t_\rho} \langle P\rangle(\neg r \supset q)$$

NB: Do not forget to draw a conclusion from the tableau.

7. **Soundness and completeness (10pt)** Consider the following tableau in K_η, which contains only one branch which we call b:

$$\begin{array}{c} \neg\neg(\Box\Diamond A \wedge \Diamond\Box B), 0 \\ \Box\Diamond A \wedge \Diamond\Box B, 0 \\ \Box\Diamond A, 0 \\ \Diamond\Box B, 0 \\ 0r1 \\ \Box B, 1 \\ \Diamond A, 1 \\ 1r2 \\ A, 2 \\ B, 2 \end{array}$$

Also consider the following interpretation I:

$$\begin{array}{rcl} W &=& \{w_0, w_1, w_2\} \\ R &=& \{\langle w_0, w_1\rangle, \langle w_2, w_0\rangle\} \\ v_{w_0}(A) = v_{w_0}(B) &=& 0 \\ v_{w_1}(A) = v_{w_1}(B) &=& 1 \\ v_{w_2}(A) = v_{w_2}(B) &=& 0 \end{array}$$

(a) Is I faithful to b? If so, provide a function $f : \mathbb{N} \to W$ that shows it is. Else, explain why not.

(b) Applying more rules to b leads to an infinite open branch b'. Is I the interpretation induced by b'? Explain your answer by discussing W, R, and v.

8. **First-order modal tableau, variable domain (10 pt)** By constructing a suitable tableau, determine whether the following is valid in VK. If the inference is invalid, provide a counter-model.

$$\Box\forall x Ax \supset \Box\forall x Bx \vdash_{VK} \Box\forall x(Ax \supset Bx)$$

NB: Do not forget to draw a conclusion from the tableau.

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9. **Default logic (10 pt)** Consider the following set of default rules:

$$D = \left\{ \delta_1 = \frac{p : q}{q}, \qquad \delta_2 = \frac{p : \neg q}{r}, \qquad \delta_3 = \frac{q \vee r : \neg q}{\neg q} \right\},$$

and initial set of facts:

$$W = \{p\}.$$

This exercise is about the default theory $T = (W, D)$.

(a) Draw the process tree of the default theory (W, D).

(b) Is p a skeptical consequence of this theory? Explain your answer.

(c) Is q a credulous consequence of this theory? Explain your answer.

(d) When does a theory have more skeptical than credulous consequences?

Bonus; 10 pt For propositional logic, show without using soundness and completeness that

$$\Sigma, A \vdash B \iff \Sigma \vdash A \supset B$$

where Σ is a finite set of premises.
Hint. Use the fact that the result of a tableau test is indifferent to the order in which one lists the premises of the argument and applies the tableau rules.